OPOLE TECHNICAL UNIVERSITY
Department of Computer Science, Electrical Engineering and Automatic Control

MMANA and Ansoft Designer aided antenna design applications and FDTD Method

AVSAR ASAN, B.Sc
October 2007

Supervisor:
MGR.INŻ. ANDRZEJ WAINDOK

Contents

1 Application With MMANA ... 3

1.1 Programme Description and Terms ... 3

1.2 Ordinary Half-Wave Diple Antennas ... 5

1.2.1 Model and Calculations ... 6

1.2.2 Design Parameters, diagrams and Optimization ... 6

1.3 Yagi Antennas ... 12

1.3.1 Theory ... 12

1.3.2 Model and Calculations ... 13

1.3.3 Design Parameters, diagrams and Optimization ... 14

2 Application with Ansoft Designer ... 18

2.1 Programme Description ... 18

2.1.1 Model Setup ... 18

2.1.2 Port-Excitation setup ... 18

2.1.3 Analysis of setup and meshing ... 18

2.2 High-Frequency microstrip patch antenna Design ... 19

2.3 Contributed Electromagnetics Design ... 21

2.3.1 Maxwell's Equations ... 21

2.3.2 Boundary Conditions ... 23

3 The FDTD Method ... 25

3.1 Finite Difference Formulation ... 25

3.1.1 Finite Difference Formulation Application Example ... 26

3.2 FDTD Method Description and 2D modelling foundations ... 28

3.3 Stability Conditions ... 31

4 Table of Figures ... 32

5 REFERENCES ... 33

1 Application With MMANA

1.1 Programme Description and Terms

MMANA program is an antenna analyzing-tool based on moment-method. It can model a wide range of antenna types, calculate radiation patterns, power gains, front-to-back ratios, feed impedances, bandwidths, the effects of loading inductors, capacitors and resistors, the effects of resonant traps, the effects of some types of transmission lines, and other things of interest to anyone interested in antennas.

The results obtained exhibit a frequency bias, which gets worse as the wire diameter increases. The results obtained are not correct for the frequency specified, but are correct for a slightly lower frequency. Antenna Model handles this with an algorithm modification.

Antenna Frequency

We could model an antenna for any frequency, but the purpose of this demonstration leave the frequency set to the 14.050 MHz MMANA default frequency.

X and Y Element Directions

Antenna elements can be described in terms of X and Y directions. X and Y are any two arbitrary directions in azimuth that are orthogonal (at right angles) to each other. We could decide that the X direction is the direction between two oak trees that are going to support an antenna wire and that the Y direction is the azimuth direction that will be at right-angles to that wire.

Z Element Direction

The Z element direction is always the direction that is normal to earth (in other words, orthogonal to a line that is tangent to earth or vertical to the local earth surface).

X1, Y1, Z1, X2, Y2, Z2 are the coordinate elements.

SEG , a code number that specifies how the antenna element is to be segmented when calculating antenna characteristics.

R , the radius of an antenna element.

PULSE, PHASE DG ,VOLT are related with the source options.

VIEW, in this part you can see your antenna physically.

SWR - standing wave ratio, is a measure of how efficiently your radio is radiating the energy it produces when you transmit.

Decibel (dB), unit of measure of loss or gain. Gain has a positive value, loss has a negative value, and is = 10*log(Pout/Pin)

Antenna Gain, The relative increase in radiation at the maximum point expressed as a value in dB above a standard, in this case the basic antenna, a ½-wavelength dipole (as in Two-Poles) by which all other antennas are measured. The reference is known as 0dBD (zero decibel referenced to dipole).

There is a second 'reference' used in antenna gain figures but is used to simply give an antenna a higher gain figure than what is truly achieved.

It is known as dBi and represents the gain of an antenna with respect to an imaginary isotropic antenna - one that radiates equally in a spherical pattern (equal in all directions). It increases the antenna gain figure by 2.14dB, this being the 'gain' of a dipole over an isotropic antenna is being formulated in Formula-1 as :

$$Gh\ dBi = 2.14dB + Gh\ dBd \qquad \text{(Equation-1)}$$

Front-Back Ratio(F/B), The driven element of most directional antennas is a dipole with the classic "doughnut" shape radiation pattern perpendicular to its axis. The idea, as shown, is to take this doughnut radiation pattern and squeeze it in to a beam off the front of the antenna. The reflector is usually just a single rod, maybe a collection of them. Even if a bunch, the reflector is not going to stop every scrap of energy from escaping between the cracks! Some will be radiated towards the rear (or, in the case of reception, bypass the reflector and be intercepted by the dipole).

1.2 Ordinary Half-Wave Diple Antennas

The equation-2 as stated below, belongs to a general linear antenna.And specificly an ordinary half (wave) dipole antenna. From there we can get the general expression for half-wave dipole antenna corresponding to L=λ/2

$$J(r) = \hat{z} I(z)\delta(x)\delta(y) \quad \text{(Equation-2)}$$

Theory continue from thereon;

$$I(z) = I\cos(kz) \quad \text{(Equation-3)}$$

And Gain;

$$g(\theta) = \frac{\cos^2(0.5\pi\cos\theta)}{\sin^2\theta} \quad \text{(Equation-4)}$$

Note that maximum gain occurs in θ = π/2 . And radiation Power as (Equation-5) is found ;

$$P_{rad} = U_{\max}\Delta\Omega = \frac{\eta\left|I^2\right|}{8\pi^2} 7.6581 = \frac{1}{2} R_{rad} \left|I^2\right|$$

Also noting that the radiation intensity occurs at $\theta = \frac{\pi}{2}$

1.2.1 Model and Calculations

In this example at 100Mhz;

Length[m]= 150 / frequency [mHz]

Lenght=150 / 100 = 1.5 m

X1= -0.75 X2=0.75 R=4 mm

Seg = -1 (means automaticly increasing/decreasing segment number note that the higher frequency goes the more segment division)

Pulse : w 1 c

( ) shows how many segments should source shift , the choices are center, b for left end and c for right end

1.2.2 Design Parameters, diagrams and Optimization

No.	F (MHz)	R (Ohm)	jX (Ohm)	SWR 75.0	Gh dBd	Ga dBi	F/B dB	Elev.	Ground	Add H.
2	100.0	72.477	0.925	1.04	-0.03	2.12	-18.08	90.0	Free	---
1	100.0	87.507	47.608	1.82	0.02	2.17	-147.68	90.0	Free	---

Figure 1.1: Free Space values

First line in Fig.1.1 is without optimizing. As optimization, I have changed just and Y1 coordinate of the dipole. Fig.1.2 a) shows before optimizing the results, and as mentioned above in optimization, those results have been achieved only by changing Y coordinate of the antenna.

We could also change the frequency range and source position. But also sometimes we have to work at a stated, at a fix frequency.When we change the positions of the antenna that would be an advantage all the time.

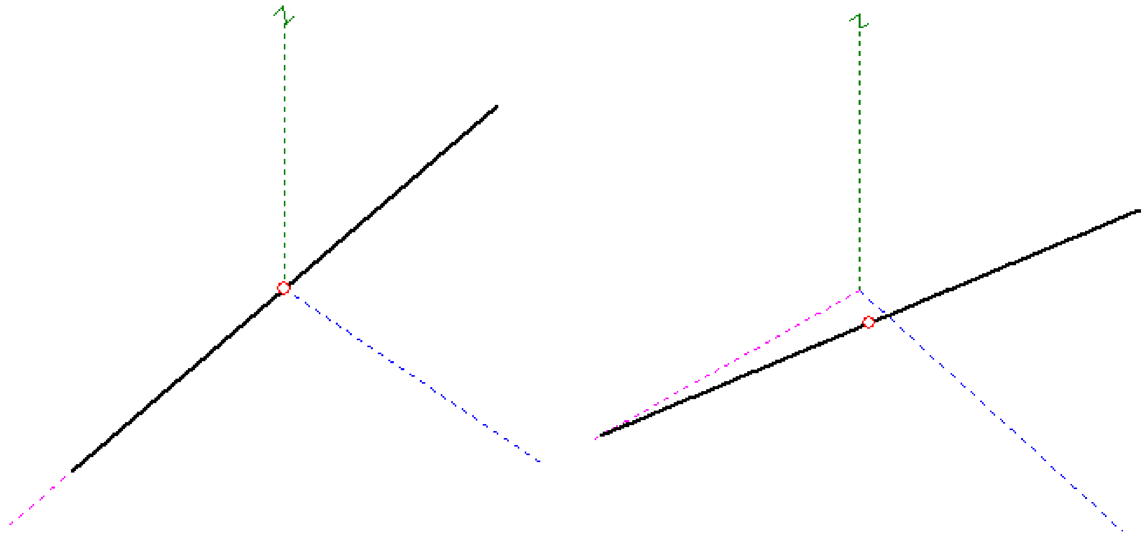

Figure 1.2: a)Before Optimization b) After Optimization

No.	F (MHz)	R (Ohm)	jX (Ohm)	SWR 75.0	Gh dBd	Ga dBi	F/B dB	Elev.	Ground	Add H.
2	100.0	72.311	-0.454	1.04	---	6.14	-1.11	67.0	Real	9.0
1	100.0	90.823	48.539	1.84	---	5.92	-0.28	78.0	Real	10.0

Figure 1.3: Real Space values

And in real space I did the optimizations in the cu wire,via height an source.Usually we started with a 1V source but here after optimization voltage source is 1,1V and the height is 10m.

So; next section is going to be revealing the plots after optimization

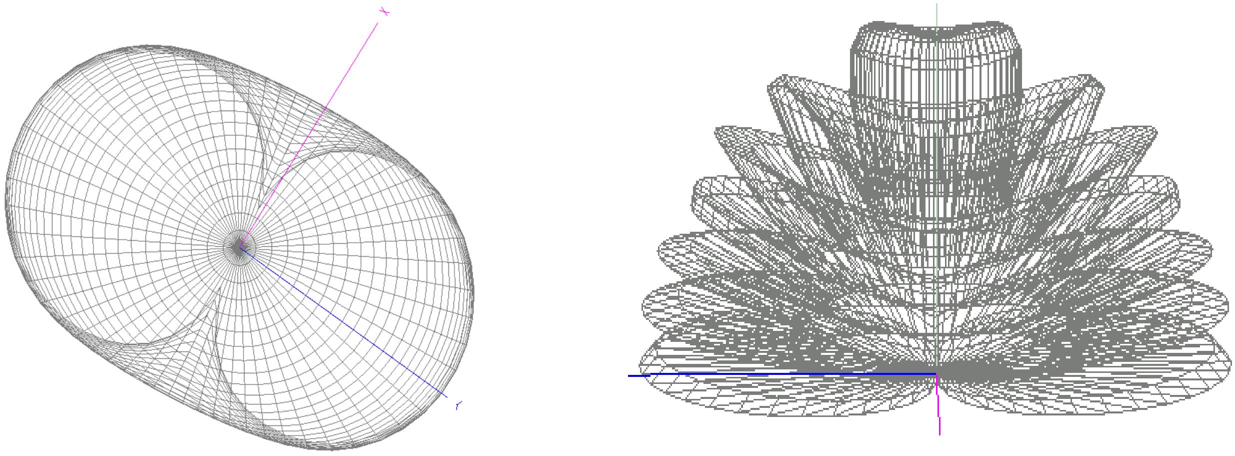

Figure 1.4: 3D View a) Free Space b) Real Space

This difference comes from,reflection property of the ground.

Effect of ground to the antennas:

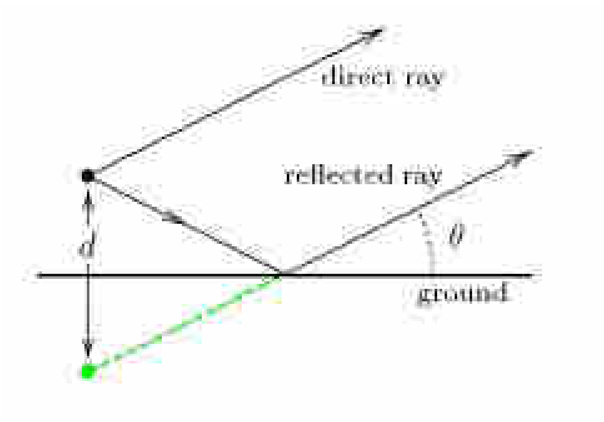

Figure 1.5: Reflection property of ground

At frequencies used in antennas, the ground behaves mainly as a dielectric. The conductivity of ground at these frequencies is negligible. When an electromagnetic wave arrives at the surface of an object, two waves are created: one enters the dielectric and the other is reflected. Illustration can be seen in Fig 1.5

If the object is a conductor, the transmitted wave is negligible and the reflected wave has almost the same amplitude as the incident one. When the object is a dielectric, the fraction reflected depends (among others things) on the angle of incidence. When the angle of incidence is small (that is, the wave arrives almost perpendicularly) most of the energy traverses the surface and very little is reflected. When the angle of incidence is near 90° (grazing incidence) almost all the wave is reflected.

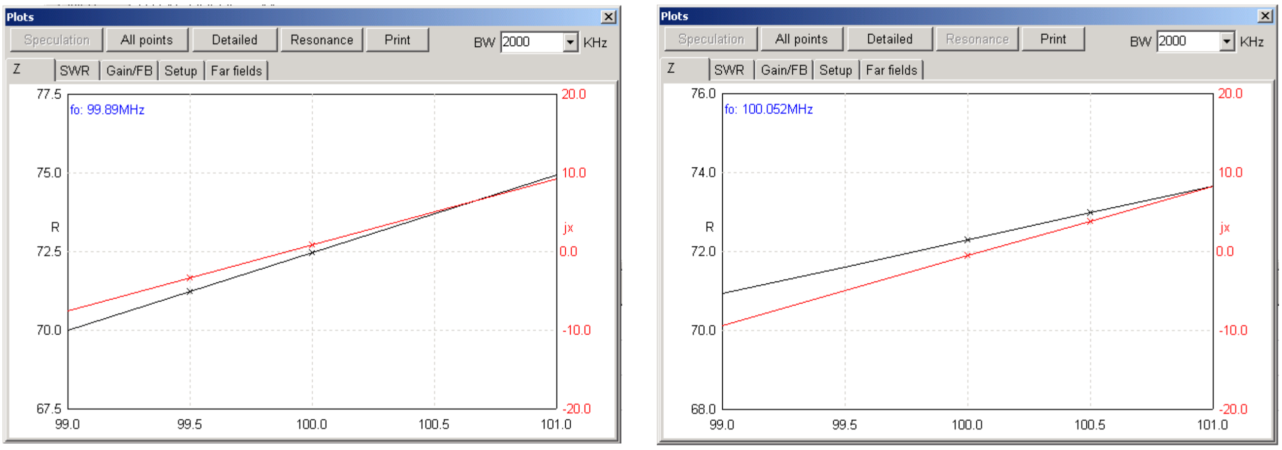

Figure 1.6: Empedances a) Free Space b) Real Space

In real-space imaginary part of impedence (jx) is lower and resistive part of the impedance(R) higher.This difference comes from antenna environment; Placing a dipole antenna some height above ground less than about 2 wavelengths (2λ) will effect both the natural feedpoint impedance and the bandwidth at that impedance. Ground clutter in the near field of the antenna will affect both in ways that are for practical purpose unpredictable.

The impedance of an antenna also affected by the surroundings and the radiation leaving the antenna through the back lobe gets reflected on the ground and is picked up again by the back lobe and adds to the power reflected by the antenna thereby changing the impedance. The empedance measurement results of the designed antenna can be seen in Fig 1.6

Another main point is SWR. As an electro-magnetic wave travels through the different parts of the antenna system it may encounter differences in impedance like I mentioned above .And at each interface, some fraction of the wave's energy will reflect back to the source, forming a standing wave in the feed line. . The SWR measurement results of the designed antenna can be seen in Fig 1.7

The ratio of maximum power to minimum power in the wave can be measured and is called the standing wave ratio (SWR). Minimizing impedance at each interface will reduce SWR in real space and maximize power transfer through the direction of the antenna . High SWR on a line will produce significant additional loss on antenna.

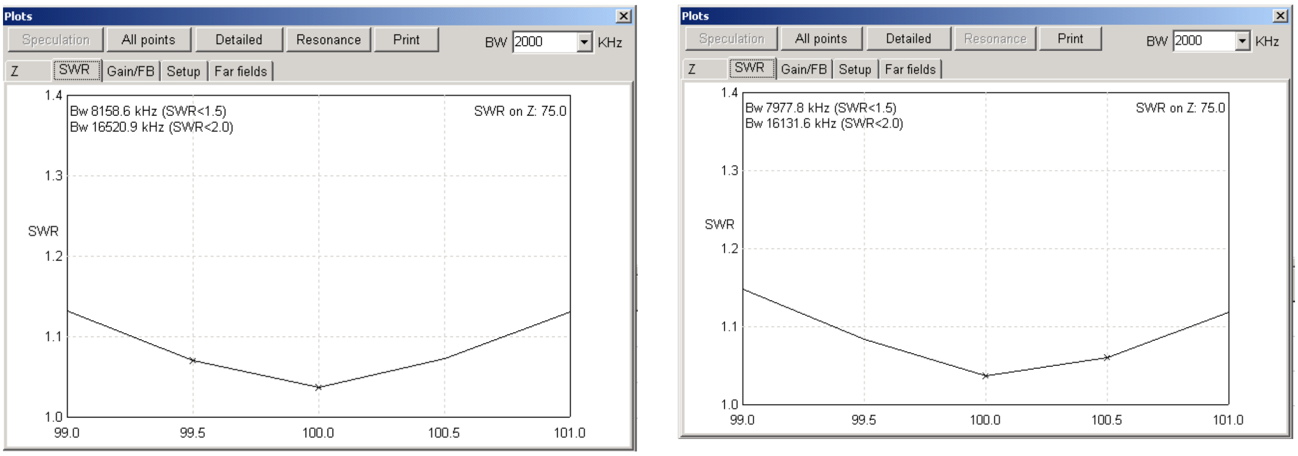

Figure 1.7: SWR a) Free Space b) Real Space

In F/B Gain; The results are listed in Figure 1.8. This difference comes from the environment of antenna and relationship between gain and F/B ratio . Gain as a parameter measures the directionality of a given antenna. Specifically, the Gain of an antenna is defined as the ratio of the intensity (power per unit surface) radiated by the antenna in a given direction at an arbitrary distance divided by the intensity radiated at the same distance like isotropic antennas. If an antenna has a greater than one gain in some directions, it must have a less than one gain in other directions since energy is conserved by the antenna. Antenna with a low gain emits radiation with about the same power in all directions.Real space has an opposite situation that's why in real space both F/B ratio and gain is higher.

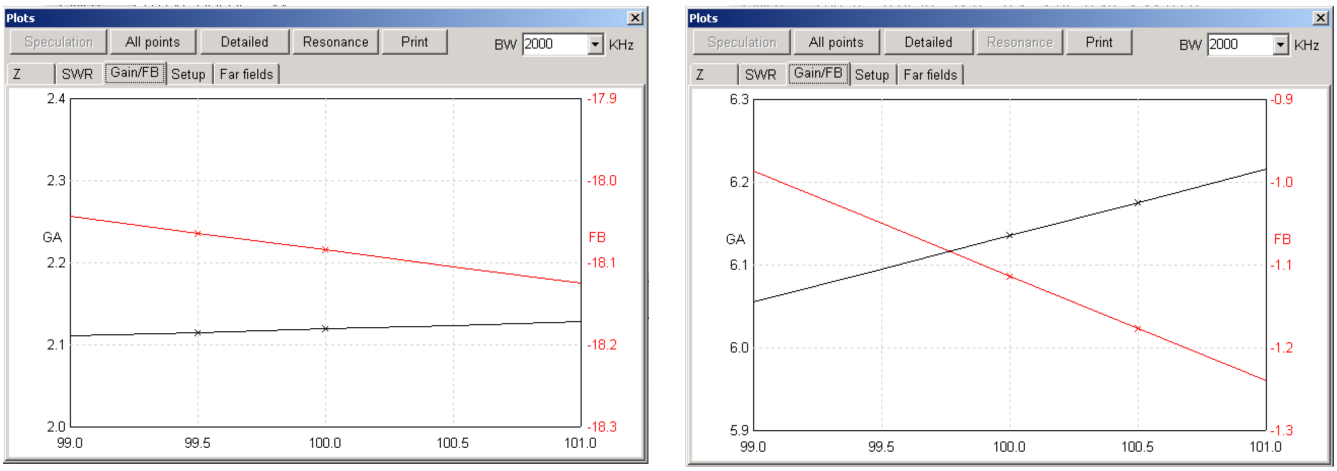

Figure 1.8: F/B Gain a) Free Space b) Real Space

Far field measurements comes with a major diference. This difference comes from partially-high-gain availability in real space.It activates vary-direction-radiating habit of antenna so that antenna radiates elliptic on real space.And difference comes also from Parasitic elements.They are usually metallic conductive structures which re-radiate into real space impinging electromagnetic radiation coming from or going to the active antenna. Element which does not have any wired input instead, it absorbs waves radiated from active antenna element in proximity, and re-radiates it in phase with the active element so that it adds to the total transmitted signal. This changes the antenna radiation pattern and beam width. Results can be seen in Fig 1.9

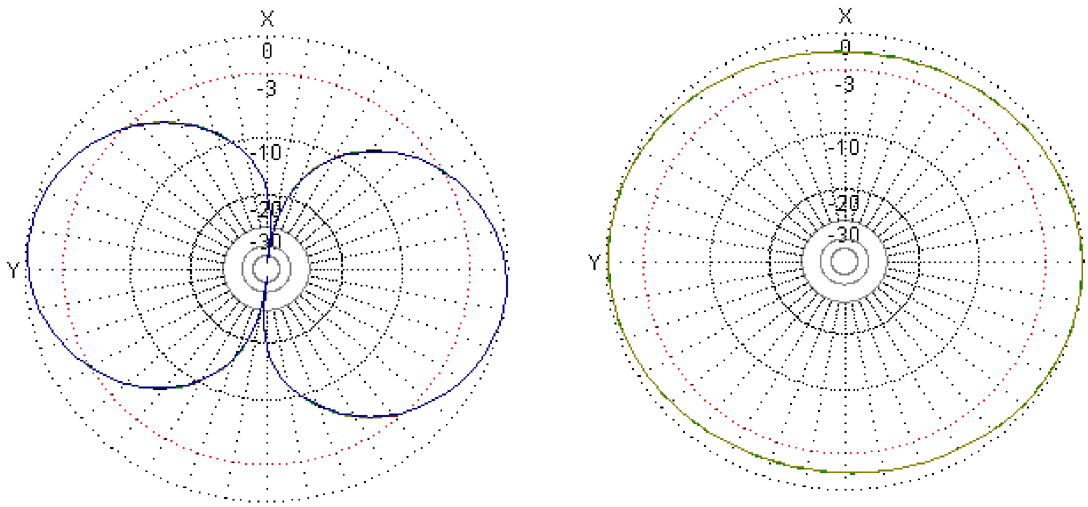

Figure 1.9:   Far Fields   a) Free Space   b) Real Space

1.3 Yagi Antennas

1.3.1 Theory

A Yagi anntenna is a unidirectional antenna commonly used in communications. The driven element of a Yagi is the equivalent of a center-fed, half-wave dipole antenna. Parallel to thedriven element, and approximately 0.2 to 0.5 wavelength on either side of it, are straight rods or wires called *reflectors* and *directors*.

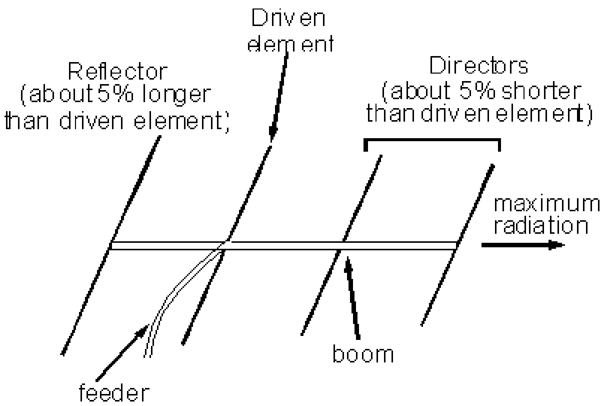

Figure 1.10: Yagi Antenna

A reflector is placed behind the driven element and is slightly longer than 1/2 wavelength; a director is placed in front of the driven element and is slightly shorter than 1/2 wavelength. A typical Yagi has one reflector and one or more directors. The antenna propagates electromagnetic field energy in the direction running from the driven element toward the director(s), and is most sensitive to incoming electromagnetic field energy in this same direction The antenna components can be seen in Fig 1.10.

The Yagi antenna not only has a unidirectional radiation and response pattern, but it concentrates the radiation and response. The more directors a Yagi has, the greater the so-called *forward gain*. As more directors are added to a Yagi, it becomes longer. Some Yagi antennas have as many as 10 or even 12 directors in addition to the driven element and one reflector. Long Yagis are rarely used below 50 MHz, because at these frequencies the structure becomes physically unwieldy.

1.3.2 Model and Calculations

Practically from the table given ;

Frequency	Transverse dimension ($\lambda/2$)	Lenght (3 elements)	Length (5 elements)	Length (15 elements)
30MHz	5 m	6 m	13 m	47 m
100MHz	1.5 m	1.8 m	3.9 m	14 m
300MHz	50 cm	60 cm	1.3 m	4.7 m
1 GHz	15 cm	18 cm	39 cm	1.4 m
3 GHz	5 cm	6 cm	13 cm	47 cm

Figure 1.11: A practical calculation table for Yagi Antennas

Reflector lenght(L_R) = 148/f

Driven Element lenght(L_D) = 143/f

Director lenght (L_{DIR}) = 138/f

h1 <distance between driven element and director(s)>…………………….. $h_1 = \lambda . 0,13$

h2 < distance between director(s) and reflector> ……………………………..$h_2 = \lambda . 0.15$

From this table and equalities given , in the example;

f = 100 MHz that means; $\lambda/2$ = 1.5 m from the table.

L_R = 148 / 100 = 1.48 m

L_D = 143 / 100 = 1.43 m

L_{DIR} = 138 / 100 = 1.38 m

h1 = λ . 0,13 m = 0,39 m

h2 = λ . 0,15 m = 0.45 m and R(mm) = 2 and 20 segment. Please see Fig 1.12

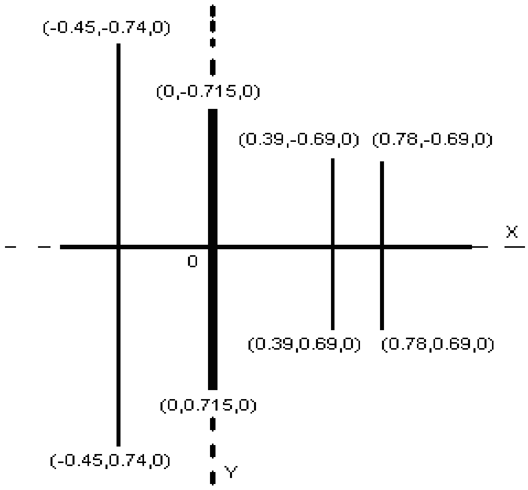

Figure 1.12: An illustration to calculated Yagi antenna

1.3.3 Design Parameters, diagrams and Optimization

No.	F (MHz)	R (Ohm)	jX (Ohm)	SWR 75	Gh dBd	Ga dBi	F/B dB	Elev.	Ground	Add H.
2	100.0	65.64	-1.441	1.14	6.86	9.01	8.55	---	Free	---
1	100.0	17.8	14.11	4.37	7.27	9.42	10.18	---	Free	---

Figure 1.13: Free space Yagi Antenna values

No.	F (MHz)	R (Ohm)	jX (Ohm)	SWR 75	Gh dBd	Ga dBi	F/B dB	Elev.	Ground	Add H.
2	100.0	65.35	-1.3	1.15	---	14.97	8.63	1.7	Real	25.5
1	100.0	17.76	14.09	4.38	---	15.36	10.27	1.7	Real	24.5

Figure 1.14: Real space Yagi Antenna values

Figure 1.13 is without optimizing. In optimization process I chose elements 1,2,3 and 4 and change their width via Y-axis.And voltage source.It is no more 1V, its 0.98V after optimization process. Also Figure 1.13 shows the situation without optimizing.Again in the optimizing process I have chosen the elements 1,2,3 and 4 to change their width via Y-axis and height to approach the perfect situation SWR = 1.

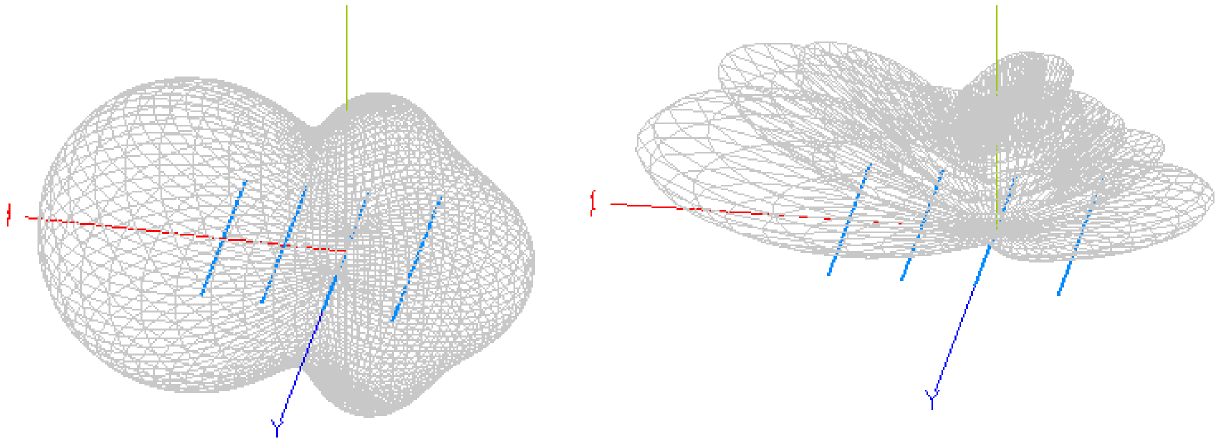

Figure 1.15: 3D view a) Free Space b) Real Space

Difference (like Fig 1.4) comes from the waves reflected from ground. That certainly proves the reflection property of the ground. In terms of Yagi antenna 3D view please see Fig 1.15

In dipole antennas, as it is said that the radiation leaving the antenna through the back lobe gets reflected on the ground thereby changing the impedance.And this is only depends on the direction of the antenna and where the ground is.So here in Yagi antenna we accept that ground lies on x-axis,that means direction of the antenna and the ground has a 180° angle and ground cannot reflect the backside wave.That means almost-no-change in between free space and real space.

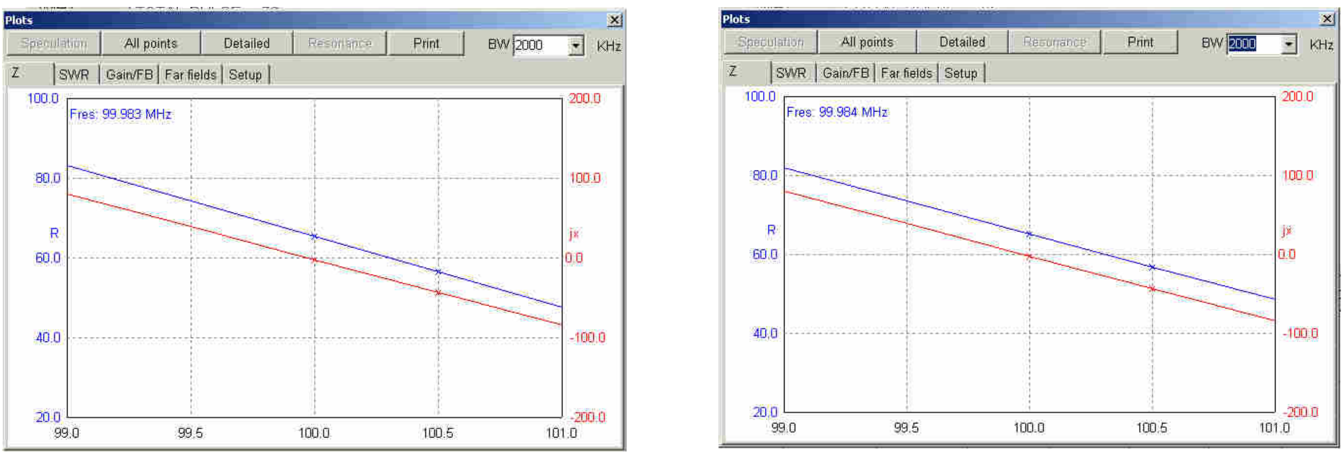

Figure 1.16: Empedances a) Free Space b) Real Space

As I mentioned in dipol antennas section; the wave reflects back,because of the ground,forms a standing wave in the feed line.Here there is no ground reflecting,that exactly means no standing wave in the feed line. Consequently no SWR changes except for the average losses.See Fig 1.17

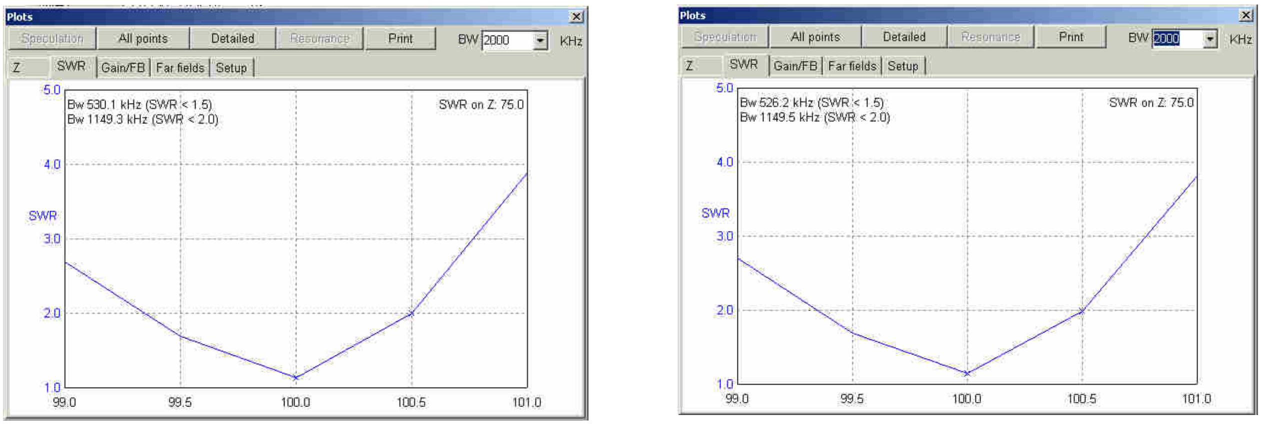

Figure 1.17: SWR a) Free Space b) Real Space

Same like the previous antenna listing, gain is tied to the environment and also from the relationship between gain and F/B ratio . Antenna with a low gain emits radiation with about the same power in all directions.Real space has an opposite situation that's why in real space both F/B ratio and gain is higher. See Fig 1.18

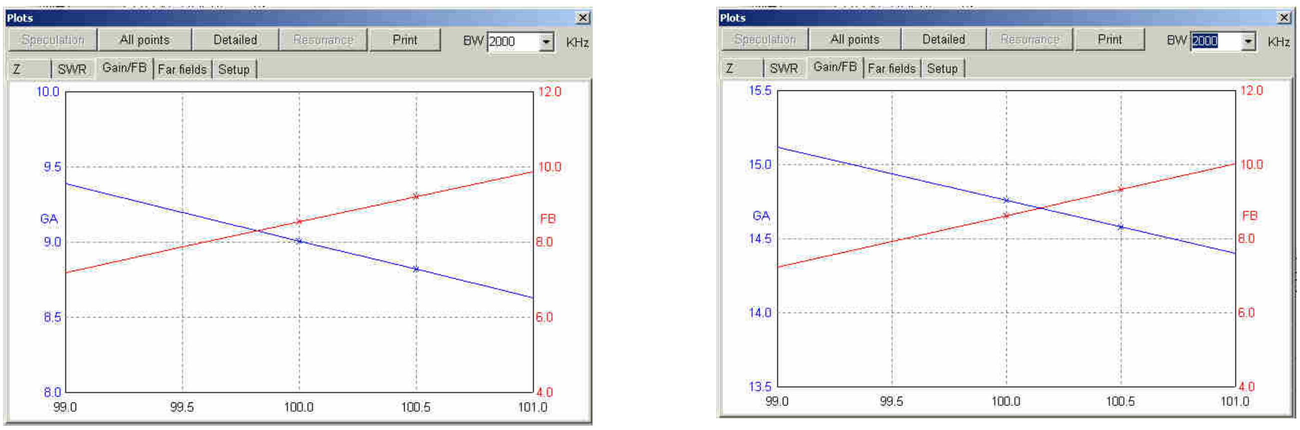

Figure 1.18: Gain F/B a) Free Space b) Real Space

Parasitic conductive structures cannot absorbe the waves reflected backwards because reflector in Yagi reflects almost all of it.So in real space there will be no phase addition to the radiation pattern by parasitic elements.Except for the average losses that means almost the same pattern both in real space and free space.See Fig 1.19

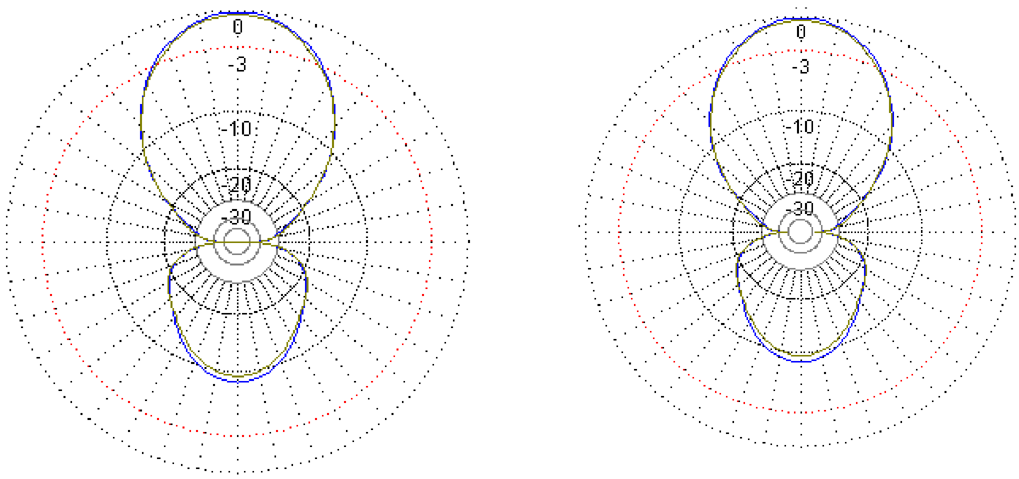

Figure 1.19: Gain F/B a) Free Space b) Real Space

2 Application with Ansoft Designer

2.1 Programme Description

Ansoft Designer,in general meaning,a simulation programme based on MoM.And enables engineers to design, optimize, and validate component circuit, and system performance long before building a prototype in hardware.

Ansoft Designer combines system-level behavioral base-band and RF blocks with transistor level detail for analog circuits and3D full-wave electromagnetic detail at the component level to provide accurate board-level simulations of EVM, BER, ACPR, IP3, and PAE.

2.1.1 Model Setup

Rectangular substrate antenna consists of two rectangular part joined together and ported a source on. Note that a quarter-wave length transformer was used to match the patch to a 50 Ohm feed line. In this example of mine, a 1.8 GHz microstrip patch antenna fed by a microstrip line on a 2.2 permittivity substrate is studied.Measures(mm) are given in Fig 2.1

2.1.2 Port-Excitation setup

Is fixed to be 1mA in current and has a 0dg phase. In order to get an accurate result, the waveport has to be defined properly; if it is too small the field will be truncated (characteristic impedance will be incorrectly calculated) and if it is too large a waveguide mode may appear.

2.1.3 Analysis of setup and meshing

We used a fixed mesh at 1.8GHz. I know that adaptively refines the mesh: the more passes we have the more accurate solution is expected. Big frequency choice needs more CPU time for a more accurate solution and also more elements.So that’s why in student version of Ansoft Designer mash in high frequencies will not work due to the lack of element.Elements we use in SV are limited.

In sweep adjustment I chose discrete current and also activate the generate surface current in order to have far-field-realization graphics.Also we have to specify frequency sweep.For this,in my example,by linear steps of 0.1GHz,from 1GHz to 20GHz.

2.2 High-Frequency microstrip patch antenna Design

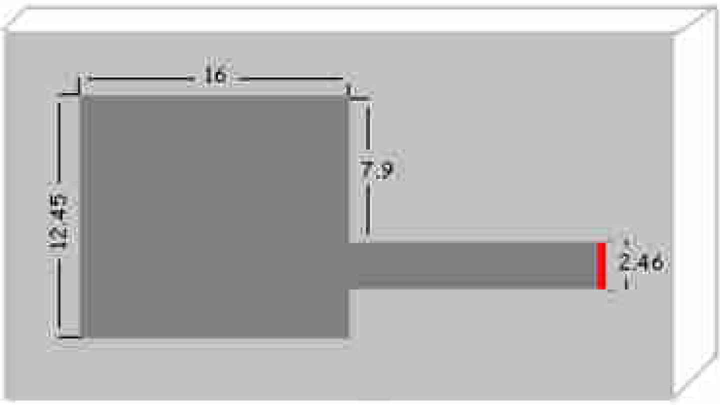

Figure 2.1: Microstrip patch antenna

The microstrip patch antenna is a resonant antenna for narrow band microwave wireless links that require semi-hemispherical coverage. Due to its planar configuration and ease of integration with microstrip technology , the microstrip patch antenna has been heavily studied and is often used as elements for an array.

To start the calculation and simulation results of dB vs frequency for the relevant antenna have been revealed in Fig 2.2

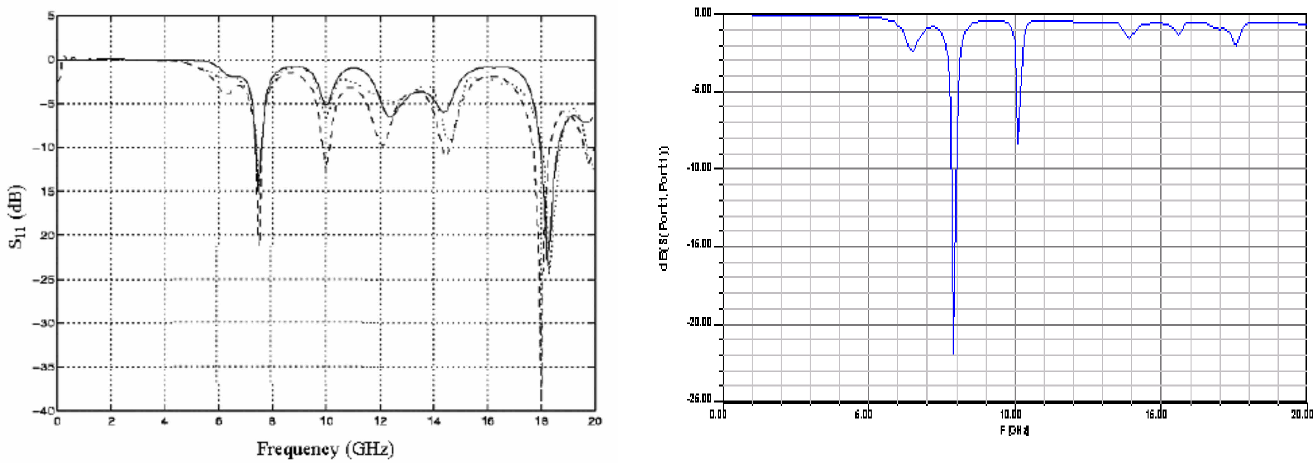

Figure 2.2: a) Calculation results b) Simulation results

Right after designing the antenna in Ansoft Designer, from the reports we have graphic dB vs GHz of port 1 in S domain. From Fig 2.2-b I can see that fundamental resonance of antenna occurs at approximately 7.8 GHz with an approximate return loss of -22.00 dB.The top face of the substrate was selected and the Electric Field Vector was plotted for 7.8 GHz.

This two graphics aresupposed to be the same, buthere you are the explanations for this;

In Fig 2.2-a , linear steps of frequency is 2GHz, In Fig 2.2-b steps are 0.1 GHz that means Fig 2.2-b is more detailed and has more exact curves.

In Fig 2.2-a , S(dB) has steps of 5dB,But in Fig 2.2-b steps are 1dB.Again that makes Fig 2.2-b more detailed in this side,too.

Ansoft Designer automatically activates surface peak currents,so that external losses which held practicly,can be seen at simulation.So,in fact,we compare the “perfect situation” without no losses in calculations and “real situation” with metallic resistive losses and surface peak currents.

Also we have the 3-D graphic of the microstrip patch antenna like in Fig 2.3.

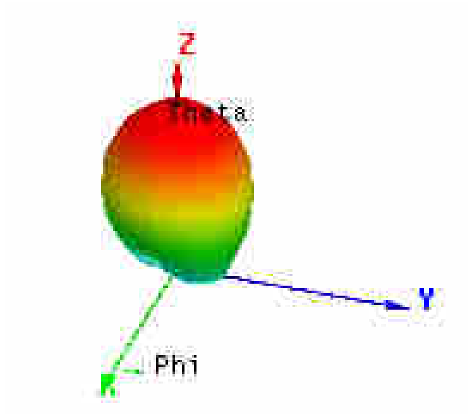

Figure 2.3: 3D graph of simulated patch antenna

Phi should start at 0 deg and stop at 90 deg with a 90 deg step size. For the three-dim pattern , the default values can be used.

2.3 Contributed Electromagnetics Design

2.3.1 Maxwell's Equations

Maxwell's equations describe all classical electromagnetic phenomena in antennas. Maxwell's equations, in fact, consists of 4 different formulas:

1-Faraday's Law of Induction $$\nabla \times E = -\frac{\partial B}{\partial t}$$

2-Ampere's Law $$\nabla \times H = J + \frac{\partial D}{\partial t}$$

3-Gauss's Law of Electric Field $$\nabla \cdot D = \rho$$

4-Gauss's Law of Magnetic Field $$\nabla \cdot B = 0$$

∇ operator expresses source or sink at a given point of electric/magnetic field.

E,H are electric and magnetic field intensities [volt/m]

D,B are electric and magnetic flux densities [coulomb/m^2]

B, also called magnetic induction.

ρ ,volume charge density

J ,electric charge density of any external charges.

Note that ρ and J may be thought of as the sources of the electromagnetic fields.

By some constitutive relations D,B are related to the field intensities E,H

$$D = \varepsilon_o E \quad \text{and} \quad B = \mu_o H$$

ε_o and μ_o are the permittivity and permeability of vacuum with numerical values.

More generally constitutive relations may inhomogeneous, anisotropic,

nonlinear,frequency dependent,or all of the above.

Here in this example we accept that materials are homogeneous,isotropic and linear.

From there if we made explicit in Maxwell's equations by usin constitutive relations and P,induced polarization and M,magnetization:

$$D = \varepsilon_o E + P \qquad\qquad B = \mu_o (H + M)$$

Respectively,now,we can express the antenna phenomena exactly by using Maxwell's equations:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_o \varepsilon_o \frac{\partial E}{\partial t} + \mu_o [J + \frac{\partial P}{\partial t} + \nabla \times M]$$

$$\nabla \cdot E = \frac{1}{\varepsilon_o} (\rho - \nabla \cdot P)$$

$$\nabla \cdot B = 0$$

2.3.2 Boundary Conditions

It's important to express boundary conditions between electromagnetic fields.Boundary conditions make a compute easily successful or unsuccessful.And to emphasis the difference between changing conditions. For the illustration of boundary conditions please see Fig 2.4

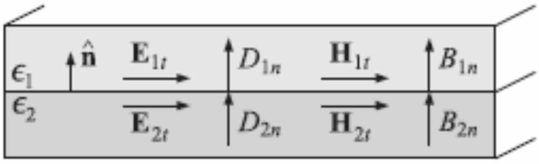

Figure 2.4: Boundary conditions

D_{1n} and D_{2n} are boundary conditions of each surface .

$\hat{n}$, unit vector normal to the boundary pointing from surface 2 to surface 1.

ρ_s and J_s are any external surface charge and surface current densities on boundary surface.

The tangential components of the E-field are continuous across the interface;the difference of the tangential components of the H-field are equal to the surface charge density;and the normal components of B(magnetic flux density) are continuous.

D_n boundary condition also depends on polarization and can be written;

$$(\varepsilon_0 E_{1n} + P_{1n}) - (\varepsilon_0 E_{2n} + P_{2n}) = \rho_s$$

$$\varepsilon_0 (E_{1n} - E_{2n}) = \rho_s - P_{1n} + P_{2n} \qquad \rho_s - P_{1n} + P_{2n} = \rho_{s,total}$$

Total surface charge density will be $\rho_{s,total} = \rho_s + \rho_{1s,pol} + \rho_{2s,pol}$.

And if we accept that surface charge densities are accumulating by $\hat{n}$;

consequently

$$\rho_{s,total} = P_n = \hat{n} \cdot P$$

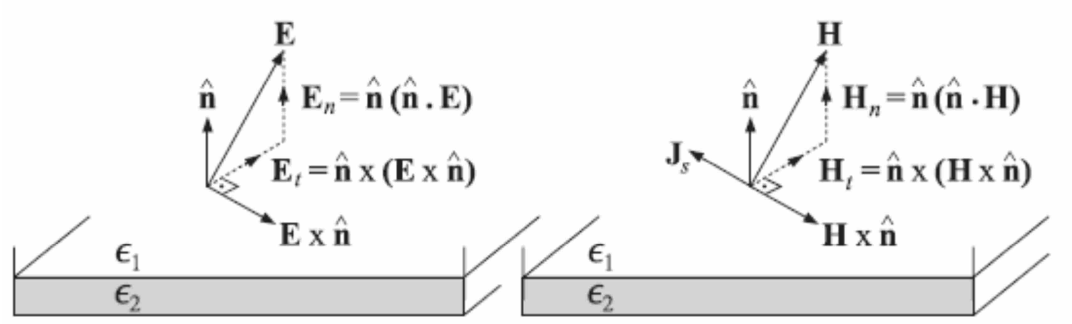

Figure 2.5: Boundary conditions with the respect of vectors in case

Using vector equation;

$$E = \hat{n} \times (E \times \hat{n}) + \hat{n}(\hat{n} \cdot E) = E_t + E_n$$

$$H = \hat{n} \times (H \times \hat{n}) + \hat{n}(\hat{n} \cdot H) = H_t + H_n$$

Then we get the certain boundary condition equations ;

$$E_{1t} - E_{2t} = 0$$

$$H_{1t} - H_{2t} = J_s \times \hat{n}$$

$$D_{1n} - D_{2n} = \rho_s$$

$$B_{1n} - B_{2n} = 0$$

3 The FDTD Method

3.1 Finite Difference Formulation

The-finite-difference method is a useful term in solving differential equations.And differential equations are used to describe real-world systems based on their mathematical models usually extracted from measurements.

To sum up main idea of finite difference method is to replace the higher order differential equations equivalently by a set of first-order equations.

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} \frac{dy}{dx} + a_n y + a_{n+1} = 0$$

In this equation we can replace;

$$y_1 = y \ , \ y_2 = \frac{dy_1}{dx} \ , \ y_3 = \frac{dy_2}{dx} \ , \ \dots\dots \ , \ y_n = \frac{dy_{n-1}}{dx}$$

And we get:

$$\frac{dy_n}{dx} + a_1 y_n + \dots + a_{n-1} y_2 + a_n y_1 + a_{n+1} = 0$$

When solving the field distribution within a given geometry,the usual zero initial condition combined with boundary conditions will decide a unique solution to the problem.Physically all the

real systems start from a certain initial condition.

$$f\left(x_0 + \Delta x\right) = f\left(x_0\right) + (\Delta x)\frac{d}{dx} f\left(x\right) \Big|_{x_0} + (\Delta x)^2 \frac{d^2}{dx^2} f(x) \Big|_{x_0} + \dots\dots$$

From forward difference

$$\frac{d}{dx}f(x)\,\Big|_{x_0} = f_F{}'(x_0) = \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

Then we get

$$\frac{d}{dx}f(x)\,\Big|_{x_0} = f_F{}'(x_0) - \frac{\Delta x}{2!}\frac{d^2}{dx^2}f(x)\,\Big|_{x_0} - \frac{(\Delta x)^2}{3!}\frac{d^3}{dx^3}f(x)\,\Big|_{x_0}$$

$$\frac{d}{dx}f(x)\,\Big|_{x_0} = \frac{f(x_0)-f(x_0+\Delta x)}{\Delta x} + \frac{(\Delta x)^2}{2!}\frac{d^2}{dx^2}f(x)\,\Big|_{x_0} + \frac{(\Delta x)^2}{3!}\frac{d^3}{dx^3}f(x)\,\Big|_{x_0} + \ldots$$

So we can get derivative in general meaning from here;

$$\frac{d}{dx}f(x)\,\Big|_{x_0} = f_F{}'(x_0) + \frac{(\Delta x)}{2!}\frac{d^2}{dx^2}f(x)\,\Big|_{x_0} + \frac{(\Delta x)^2}{3!}\frac{d^3}{dx^3}f(x)\,\Big|_{x_0} + \ldots$$

3.1.1 Finite Difference Formulation Application Example

For two dimensional Poisson’s equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{-\rho(x,y)}{\varepsilon_0}$$

Note : second order forward difference

$$\frac{\partial^2}{\partial x^2} f(x) = f_c{}'(x_0) = \frac{f(x_0+\Delta x) + f(x_0-\Delta x) - 2f(x_0)}{\Delta x^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{u_{i-1,j} + u_{i+1,j} - 2u_{ij}}{\Delta x^2} \quad \text{and} \quad \frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j-1} + u_{i,j+1} - 2u_{ij}}{\Delta y^2}$$

With source charges equations turns into;

$$\frac{u_{i-1,j} + u_{i+1,j} - 2u_{ij}}{\Delta x^2} + \frac{u_{i,j-1} + u_{i,j+1} - 2u_{ij}}{\Delta y^2} = -\frac{\mp e_0}{\varepsilon_0}$$

Respectively at boundaries x=0, y=0 , voltages are fixed to zero

$$\Delta x = \Delta y = 0$$

$$u_{i,j} = 0$$

$$u_{i-1,j} + u_{i+1,j} - 2u_{ij} + u_{i,j-1} + u_{i,j+1} - 2u_{ij} = -\frac{\mp e_0}{\varepsilon_0}$$

$$4u_{ij} = u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} + \frac{\mp e_0}{\varepsilon_0}$$

As conclusion we get ; $u_{ij} = \dfrac{u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}}{4} + \dfrac{\mp e_0 \Delta^2}{4\varepsilon_0}$

3.2 FDTD Method Description and 2D modelling foundations

In section 2.2.1 , I have already mentioned about Maxwell's equation.But before I didn't take an attention to the time especially when space and time together,2-dim Maxwell equations are not enough to express all conditions.So as like finite difference formulation I have to express them via partial-differential equations.

Note to divergence theorem : $\iiint_V (\nabla \times F) dV = \iint_{\partial V} F.n.dS$

Note to Kelvin-stokes theorem : $\int_{\Sigma} (\nabla \times F) d\Sigma = \oint_{\partial \Sigma} F dr$ Respectively;

$\nabla \times E = -\frac{\partial B}{\partial t}$ turns into $\oint_C E dl = \int_S (\nabla \times E) dA = -\frac{d}{dt} \int_S B dA$ ....(i)

$\nabla \times H = J + \frac{\partial D}{\partial t}$ turns into $\oint_C H dl = \int_S (\nabla \times H) dA = \int_S J dA + \frac{d}{dt} \int_S D dA$ ....(ii)

Operation of curl (rot) is the ratio of rotation.And can be expressed ; $rot(F) = \vec{\nabla} \times \vec{F}$

When expanded to 3-dim Cartesian coordinates:

$$\begin{bmatrix} \dfrac{\partial F_z}{\partial y} - \dfrac{\partial F_y}{\partial z} \\ \dfrac{\partial F_x}{\partial z} - \dfrac{\partial F_z}{\partial x} \\ \dfrac{\partial F_y}{\partial x} - \dfrac{\partial Fx}{\partial y} \end{bmatrix}$$

Transforms (i) and (ii) can be written in 3-dim Cartesian coordinates;

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t} \qquad \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x + \varepsilon \frac{\partial \mathrm{E}_x}{\partial t}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu \frac{\partial H_y}{\partial t} \qquad \frac{\partial Hx}{\partial z} - \frac{\partial H_z}{\partial x} = Jy + \varepsilon \frac{\partial \mathrm{E}_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu \frac{\partial Hz}{\partial t} \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z + \varepsilon \frac{\partial \mathrm{E}_z}{\partial t}$$

So, from here when an electromagnetic field has a constant distribution in one of three directions than it becomes 2-dimensional.

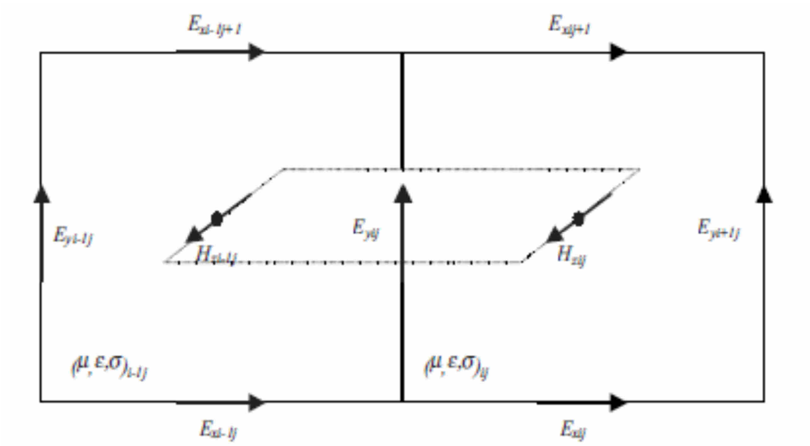

Figure 3.1: Two adjacent 2D Yee cells for TM mode FDTD formulation.

Lets say radiated electromagnetic quantities are independent from z-axis.To represent all the grids, we I will use the notation $(x_i, y_j, t_n) = (i\Delta x,\ j\Delta y, n\Delta t)$ Where i,j,n are half integers in 2-dim formulation.

In the lack of z-axis problem is transverse magnetic (TM) polarized.For a TM distribution Ampere's and Faraday's Laws reduce to;

$$\frac{\partial E_z}{\partial y} = -\mu \frac{\partial H_x}{\partial t} \quad , \quad \frac{\partial E_z}{\partial x} = \mu \frac{\partial H_y}{\partial t} \quad , \quad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \varepsilon \frac{\partial \mathrm{E}_z}{\partial t}$$

If we difference these equations in the reference of 2-dim Yee cell in Fig 3.1

$$\frac{E^{n}_{zi,j+1} - E^{n}_{zi,j}}{\Delta y} = -\mu \frac{H^{n+1/2}_{xi,j+1/2} - H^{n-1/2}_{xij+1/2}}{\Delta t}$$

$$\frac{E^{n}_{zi+1,j} - E^{n}_{zi,j}}{\Delta x} = \mu \frac{H^{n+1/2}_{yi,j+1/2} - H^{n-1/2}_{yi+1/2,j}}{\Delta t}$$

$$\frac{H^{n+1/2}_{yi+1/2,j} - E^{n+1/2}_{yi-1/2,j}}{\Delta x} - \frac{H^{n+1/2}_{xij+1/2} - H^{n+1/2}_{xi,j-1/2}}{\Delta y} = \varepsilon \frac{E^{n+1}_{zij} - E^{n}_{zi,j}}{\Delta t}$$

Transforming $\vec{\Im} = \rho \vec{\varepsilon}$ into Ampere's Law

Difference approximation to Faraday's law to compute H_x and Hy at the n+1/2 time step from E_z at time step n.

Then used Ampere's law to compute E_z in time step n+1

Alternating between these two steps allows the solution to propagate forward in time.

To this implement the FDTD method in practice, we need to add sources, apply stability conditions, ensure that the solution is stable.

3.3 Stability Conditions

Because FDTD method is an explicit scheme , there is a limit in the steps Δt to ensure the stability.It's given by ;

$$\Delta t \leq \frac{1}{c\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}}}$$

where ; c is propagation speed

$$CFL = c\Delta t\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}}$$

and from here may be written:

CFL < 1

CFL : Courant-Friedrichs-Lewy

4 Table of Figures

Figure 1.1: Free Space values 6

Figure 1.2: a)Before Optimization b) After Optimization 7

Figure 1.3: Real Space values 7

Figure 1.4: 3D View a) Free Space b) Real Space 8

Figure 1.5: Reflection property of ground 8

Figure 1.6: Empedances a) Free Space b) Real Space 9

Figure 1.7: SWR a) Free Space b) Real Space 10

Figure 1.8: F/B Gain a) Free Space b) Real Space 10

Figure 1.9: Far Fields a) Free Space b) Real Space 11

Figure 1.10: Yagi Antenna 12

Figure 1.11: A practical calculation table for Yagi Antennas 13

Figure 1.12: An illustration to calculated Yagi antenna 14

Figure 1.13: Free space Yagi Antenna values 14

Figure 1.14: Real space Yagi Antenna values 14

Figure 1.15: 3D view a) Free Space b) Real Space 15

Figure 1.16: Empedances a) Free Space b) Real Space 15

Figure 1.17: SWR a) Free Space b) Real Space 16

Figure 1.18: Gain F/B a) Free Space b) Real Space 16

Figure 1.19: Gain F/B a) Free Space b) Real Space 17

Figure 2.1: Microstrip patch antenna 19

Figure 2.2: a) Calculation results b) Simulation results 19

Figure 2.3: 3D graph of simulated patch antenna 20

Figure 2.4: Boundary conditions 23

Figure 2.5: Boundary conditions with the respect of vectors in case 24

Figure 3.1: Two adjacent 2D Yee cells for TM mode FDTD formulation. 29

5 REFERENCES

Yefet A, Turkel E . ''Construction of Three Dimensional Solutions for the Maxwell Equations'' NASA. Dec 1998

Yee K.S. ''2D FDTD Method'' IEEE Trans. Ant. Propag, vol. AP – 14, May 1996, pp 302 – 307

Edelvik F. ''Hybrid Solvers for the Maxwell Equations in Time – Domain'' ISSN 1104-2513 Uppsala, 2002

Dempster A G. ''New GNSS Signals: Receiver Design Challenges'' Int.Symposium on GNSS/GPS, 6-8 Dec 2004

Wang C, Chang K. ''A Novel CP Patch Antenna with a Simple Feed Structure'' IEEE 0-7803-6369-8, 2000

Bespalov A, 'Numerical Simulation of 3D Electromagnetic Scattering by Algebraic Fictitious Domain Method' Rapport Recherche Calcul Scientifique modelisation et logiciel numerique Programme 6, 1995

Horvath R. ''A review and Comment of the recent FDTD Literature from the point of View of the numerical solution fastness'' Scientific Computing Group , Eindhoven University of Tech,

Andersson U. '' Time-Domain Methods for the Maxwell Equations'' Doctoral Dissertation, KTH 2001

Josefsson L, Persson P. ''Conformal Array Antenna Theory and Design'' IEEE Press Series on Electromagnetic Wave Theory

Gupta K C, Hall P S ''Analysis and Design of Integrated Circuit–Antenna Modules'' Wiley pub, 2000

Orfanidis S J ''Electromagnetic Waves and Antennas'' , 2008

Johnson R C, ''Antenna Engineering Handbook'' McGraw Hill, 1994